IberoAmerican 2008

Salvador da Bahia, Brasil

Day 1 - *23 September 2008*

1. The integers from 1 to 2008^2 are written on each square of a 2008×2008 board. For every row and column the difference between the maximum and minimum numbers is computed. Let S be the sum of these 4016 numbers. Find the greatest possible value of S.

2. Given a triangle ABC, let r be the external bisector of $\angle ABC$. P and Q are the feet of the perpendiculars from A and C to r. If $CP \cap BA = M$ and $AQ \cap BC = N$, show that MN, r and AC concur.

3. Let $P(x) = x^3 + mx + n$ be an integer polynomial satisfying that if $P(x) - P(y)$ is divisible by 107, then $x - y$ is divisible by 107 as well, where x and y are integers. Prove that 107 divides m.

Day 2 - *24 September 2008*

4 Prove that the equation

$$x^{2008} + 2008! = 21^y$$

doesn't have solutions in integers.

5 Let ABC a triangle and X, Y and Z points at the segments BC, AC and AB, respectively.Let A', B' and C' the circuncenters of triangles AZY,BXZ,CYX, respectively.Prove that $4(A'B'C') \geq (ABC)$ with equality if and only if AA', BB' and CC' are concurrents.

Note: (XYZ) denotes the area of XYZ

6 *Biribol* is a game played between two teams of 4 people each (teams are not fixed). Find all the possible values of n for which it is possible to arrange a tournament with n players in such a way that every couple of people plays a match in opposite teams exactly once.