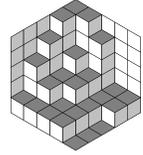




# IberoAmerican 2008

Salvador da Bahia, Brasil



---

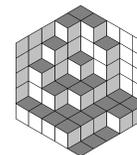
## Day 1 - 23 September 2008

- 1 The integers from 1 to  $2008^2$  are written on each square of a  $2008 \times 2008$  board. For every row and column the difference between the maximum and minimum numbers is computed. Let  $S$  be the sum of these 4016 numbers. Find the greatest possible value of  $S$ .
- 2 Given a triangle  $ABC$ , let  $r$  be the external bisector of  $\angle ABC$ .  $P$  and  $Q$  are the feet of the perpendiculars from  $A$  and  $C$  to  $r$ . If  $CP \cap BA = M$  and  $AQ \cap BC = N$ , show that  $MN$ ,  $r$  and  $AC$  concur.
- 3 Let  $P(x) = x^3 + mx + n$  be an integer polynomial satisfying that if  $P(x) - P(y)$  is divisible by 107, then  $x - y$  is divisible by 107 as well, where  $x$  and  $y$  are integers. Prove that 107 divides  $m$ .



# IberoAmerican 2008

Salvador da Bahia, Brasil



## Day 2 - 24 September 2008

- 4 Prove that the equation

$$x^{2008} + 2008! = 21^y$$

doesn't have solutions in integers.

- 5 Let  $ABC$  a triangle and  $X, Y$  and  $Z$  points at the segments  $BC, AC$  and  $AB$ , respectively. Let  $A', B'$  and  $C'$  the circumcenters of triangles  $AZY, BXZ, CYX$ , respectively. Prove that  $4(A'B'C') \geq (ABC)$  with equality if and only if  $AA', BB'$  and  $CC'$  are concurrents.

Note:  $(XYZ)$  denotes the area of  $XYZ$

- 6 *Biribol* is a game played between two teams of 4 people each (teams are not fixed). Find all the possible values of  $n$  for which it is possible to arrange a tournament with  $n$  players in such a way that every couple of people plays a match in opposite teams exactly once.